TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

UNIT-I PARTIAL DIFFERENTIAL EQUATIONS

PART-A

1. Eliminate the arbitrary constants a & b from $z = (x^2 + a)(y^2 + b)$

Answer:

$z = (x^2 + a)(y^2 + b)$

Diff partially w.r.to x & y here $p = \frac{\partial z}{\partial x}$ & $q = \frac{\partial z}{\partial y}$

$p = 2x(y^2 + b)$; $q = (x^2 + a)\,2y$

$\Rightarrow (y^2 + b) = p/2x$; $(x^2 + a) = q/2y$

$z = (p/2x)(q/2y)$

$\Rightarrow 4xyz = pq$

2. Form the PDE by eliminating the arbitrary function from $z = f(xy)$

Answer:

$z = f(xy)$, Diff partially w.r.to x & y here $p = \frac{\partial z}{\partial x}$ & $q = \frac{\partial z}{\partial y}$

$p = f'(xy).y$ $\quad$ $q = f'(xy).x$

$p/q = y/x \Rightarrow px - qy = 0$

3. Form the PDE by eliminating the constants a and b from $z = ax^n + by^n$

Answer:

$z = ax^n + by^n$,

Diff. w .r. t. x and y here $p = \frac{\partial z}{\partial x}$ & $q = \frac{\partial z}{\partial y}$

$p = nax^{n-1}$; $q = nby^{n-1}$

$$a = \frac{p}{nx^{n-1}} \quad ; b = \frac{q}{ny^{n-1}}$$

$$\therefore z = \frac{p}{nx^{n-1}} x^n + \frac{q}{ny^{n-1}} y^n$$

$$\Rightarrow nz = px + qy$$

4. Find the complete integral of p + q =pq

Answer:

Put $p = a$, $q = b$,

$p + q = pq \Rightarrow a + b = ab$

$\Rightarrow b - ab = -a \Rightarrow b = \frac{-a}{1-a} = \frac{a}{a-1}$

The complete integral is $z = ax + \frac{a}{a-1} y + c$

5. Find the solution of $\sqrt{p} + \sqrt{q} = 1$

Answer:

$z = ax+by+c$ ----(1) is the required solution

given $\sqrt{p} + \sqrt{q} = 1$ -----(2)

put $p=a$, $q = b$ in (2)

$$\sqrt{a} + \sqrt{b} = 1 \Rightarrow \sqrt{b} = 1 - \sqrt{a} \quad \Rightarrow b = (1 - \sqrt{a})^2$$

$$\therefore z = a\,x + (1 - \sqrt{a})^2\, y + c$$

6. Find the General solution of p tanx + q tany = tanz.

Answer:

$$\frac{dx}{\tan x}=\frac{dy}{\tan y}=\frac{dz}{\tan z}$$

$$\Rightarrow \cot x\,dx=\cot y\,dy=\cot z\,dz$$

$$take \int \cot x\,dx=\int \cot y\,dy$$

$$\Rightarrow \log\sin x=\log\sin y+\log c_1$$

$$\Rightarrow c_1=\frac{\sin x}{\sin y}$$

$$\int \cot y\,dy=\int \cot z dz$$

$$\Rightarrow \log\sin y=\log\sin z+\log c_2$$

$$\Rightarrow c_2=\frac{\sin y}{\sin z}$$

$$\therefore \Phi\left[\frac{\sin x}{\sin y},\frac{\sin y}{\sin z}\right]=0$$

7. Find the equation of the plane whose centre lie on the z-axis

Answer: General form of the sphere equation is $x^2+y^2+(z-c)^2=r^2$ (1)

Where ‘r’ is a constant. From (1)

2x+2(z-c) p=0 (2)

2y +2(z-c) q = 0 (3)

From (2) and (3)

$\frac{x}{p}=-\frac{y}{q}$, That is py -qx =0 which is a required PDE.

8. Find the singular integral of $z=px+qy+pq$

Answer: The complete solution is $z=ax+by+ab$

$$\frac{\partial z}{\partial a}=0=x+b \qquad ;\frac{\partial z}{\partial b}=0=y+a$$

$$\Rightarrow b=-x \quad ; \quad a=-y$$

$$\therefore z=(-y)x+(-x)y+(-y.-x)$$

$$=-xy-xy+xy$$

$$=-xy$$

$$\therefore xy+z=0$$

9**. Find the general solution of px+qy=z**

Answer:

The auxiliary equation is $\frac{dx}{x}=\frac{dy}{y}=\frac{dz}{z}$

From $\frac{dx}{x}=\frac{dy}{y}$ Integrating we get log x = log y + log c , on simplifying $\frac{x}{y}=c_1$.

$$\frac{dy}{y}=\frac{dz}{z}\Rightarrow\frac{y}{z}=c_2$$

Therefore $\varphi\left(\frac{x}{y},\frac{y}{z}\right)=0$ is general solution.

10. Find the general solution of $px^2+qy^2=z^2$

Answer:

The auxiliary equation is $\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{z^2}$

From $\frac{dx}{x^2} = \frac{dy}{y^2}$, Integrating we get $\frac{1}{y} - \frac{1}{x} = c_1$

Also $\frac{dy}{y^2} = \frac{dz}{z^2}$ Integrating we get $\frac{1}{z} - \frac{1}{y} = c_2$

Therefore $\varphi\left(\frac{1}{y}-\frac{1}{x},\ \frac{1}{z}-\frac{1}{y}\right)=0$ is general solution.

11. Solve $\left(D^2-2DD'-3D'^2\right)z=0$

Answer:

Auxiliary equation is $m^2-2m-3=0$, $(m-3)(m+1)=0$, $m=-1, m=3$

The solution is $z=f_1(y-x)+f_2(y+3x)$

12. Solve $\left(D^2-4DD'+3D'^2\right)z=e^{x+y}$

Answer: Auxiliary equation is $m^2-4m+3=0$, $(m-3)(m-1)=0$, $m=1, m=3$

The CF is $CF=f_1(y+x)+f_2(y+3x)$

$$PI=\frac{1}{D^2-4DD'+3D'^2}e^{x+y}$$ Put $D=1, D'=-1$ Denominator =0.

$$PI=\frac{x}{2D-4D'}e^{x+y} = -\frac{xe^{x+y}}{2}$$

Z=CF + PI

$$z=f_1(y+x)+f_2(y+3x)-\frac{xe^{x+y}}{2}$$

13. Find P.I $\left(D^2-4DD'+4D'^2\right)z=e^{2x-y}$

Answer:

$$PI=\frac{1}{D^2-4DD'+4D'^2}e^{2x-y}$$

Put $D=2, D'=-1$ $$PI=\frac{1}{(D-2D')^2}e^{2x-y} = \frac{1}{(2+2)^2}e^{2x-y} = \frac{e^{2x-y}}{16}$$

PART-B

1. Solve $x^2(y-z)p+y^2(z-x)q=z^2(x-y)$
2. Solve $(mz-ny)\frac{\partial z}{\partial x}+(nx-lz)\frac{\partial z}{\partial y}=ly-mx$
3. Solve $(3z-4y)p+(4x-2z)q=2y-3x$
4. Solve $\left(x^2-y^2-z^2\right)p+2xyq=2xz$
5. Solve $\left(y^2+z^2-x^2\right)p-2xyq+2zx=0$
6. Solve $(y-z)p+(z-x)q=x-y$
7. Solve $(y+z)p+(z+x)q=x+y$
8. Solve $\left(D^2-3DD'+2D'^2\right)=e^{3x-2y}+\sin(3x+2y)$

9. Solve $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial x \partial y}=\cos x \cos 2y$

10. Solve $\left(D^2+DD'-6D'^2\right)z=y\cos x$

11. Solve $\left(D^2-DD'-30D'^2\right)z=xy+e^{6x+y}$

12. Solve $\left(D^2-6DD'+5D'^2\right)z=e^x\sinh y+xy$

13. Solve $\left(D^2-4DD'+4D'^2\right)z=e^{2x+y}$

14. Solve $\left(D^3-D^2D'-DD'^2-D'^3\right)z=e^{2x+y}+\cos(x+y)$

15. Solve $z=px+qy+\sqrt{1+p^2+q^2}$

16. Solve $z=px+qy-p^2q^2$

17. Solve $z^2=1+p^2+q^2$

18. Solve $z^2(p^2x^2+q^2)=1$

19. Solve (i) $z\ (p^2-q^2)=x^2-y^2$ (ii) $z^2(p^2+q^2)=x^2+y^2$

UNIT-II FOURIER SERIES

PART-A

1. Define periodic function with example.

If a function f(x) satisfies the condition that f(x + T) = f(x), then we say f(x) is a periodic function with the period T.

Example:-

i) Sinx, cosx are periodic function with period 2π

ii) tanx is are periodic function with period π.

2. State Dirichlets condition.

(i) f(x) is single valued periodic and well defined except possibly at a Finite number of points.

(ii) f (x) has at most a finite number of finite discontinuous and no infinite Discontinuous.

(iii) f (x) has at most a finite number of maxima and minima.

3. State Euler's formula.

Answer:

$In\ (c, c+2l)$

$$f(x)=\frac{a_o}{2}+\sum a_n\cos nx+b_n\sin nx$$

$$where\ \ a_o=\frac{1}{\pi}\int_c^{c+2l} f(x)dx$$

$$a_n=\frac{1}{\pi}\int_c^{c+2l} f(x)\cos nx dx$$

$$b_n=\frac{1}{\pi}\int_c^{c+2l} f(x)\sin nx dx$$

4. Write Fourier constant formula for f(x) in the interval $(0,2\pi)$

Answer:

$$a_o = \frac{1}{\pi}\int_0^{2\pi} f(x)dx$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx dx$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx dx$$

5. In the Fourier expansion of

f(x) = $\begin{cases} 1+\frac{2x}{\pi}, -\pi < x < 0 \\ 1-\frac{2x}{\pi}, 0 < x < \pi \end{cases}$ **in (-π , π), find the value of** b_n**.**

Answer:

Since *f(-x)=f(x)* then *f(x)* is an even function. Hence b_n = 0

6. If f(x) = x^3 in –π < x < π, find the constant term of its Fourier series.

Answer:

Given f(x) = x^3 $\Rightarrow$ f(-x) = $(-x)^3$ = - x^3 = - f(x)

Hence f(x) is an odd function

The required constant term of the Fourier series = a_o = 0

7. What are the constant term a_0 and the coefficient of cosnx in the Fourier Expansion f(x) = x – x^3 in –π < x < π

Answer:

Given f(x) = x – x^3 $\Rightarrow$ f(-x) = -x - $(-x)^3$ = - [x - x^3] = - f(x)

Hence f(x) is an odd function

The required constant term of the Fourier series = a_0 = 0

8. Find the value of a_0 for f(x) = 1+x+x^2 in $(0, 2\pi)$

Answer:

$$a_o = \frac{1}{\pi}\int_0^{2\pi} f(x)dx$$

$$= \frac{1}{\pi}\int_0^{2\pi}(1+x+x^2)dx = \frac{1}{\pi}\left[x+\frac{x^2}{2}+\frac{x^3}{3}\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[2\pi+\frac{4\pi^2}{2}+\frac{8\pi^3}{3}\right] = \left[2+2\pi+\frac{8\pi^2}{3}\right]$$

9. Find b_n in the expansion of x^2 as a Fourier series in $(-\pi, \pi)$.

Answer:

Given f(x) = x^2 $\Rightarrow$ f(-x) = x^2 = f(x)

Hence f(x) is an even function

In the Fourier series b_n = 0

10. Find b_n in the expansion of xsinx a Fourier series in $(-\pi, \pi)$

Answer:

Given f(x) = xsinx $\Rightarrow$ f(-x) = (-x)sin(-x) = xsinx = f(x)

Hence f(x) is an even function

In the Fourier series b_n = 0

11. If f(x) is odd function in $(-l, l)$. What are the value of a_0 & a_n

Answer:

If f(x) is an odd function, a_o = 0, a_n = 0

12. In the Expansion f(x) = |x| as a Fourier series in (-π . π) find the value of a_o

Answer:

Given f(x) = |x| $\Rightarrow$ f(-x) = |-x| = |x| = f(x)

Hence f(x) is an even function

$$\therefore a_o = \frac{2}{\pi}\int_0^{\pi} x dx = \frac{2}{\pi}\left[\frac{x^2}{2}\right]_0^{\pi} = \frac{2}{\pi}\left[\frac{\pi^2}{2}\right] = \pi$$

13. Define R.M.S value.

If let f(x) be a function defined in the interval (a, b), then the R.M.S value of f(x) is defined by $\bar{y} = \sqrt{\frac{1}{b-a}\int_a^b [f(x)]^2 dx}$

14. State Parseval's Theorem.

Let f(x) be periodic function with period 2l defined in the interval (c, c+2l).

$\frac{1}{2l}\int_c^{c+2l} [f(x)]^2 dx = \frac{a_o^2}{4} + \frac{1}{2}\sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right)$ **Where** $a_o, a_n \& b_n$ **are Fourier constants**

15. Find the RMS value of f(x) = x², $0 < x < 1$

Answer:

Given f(x) = x^2

R.M.S value

$$\bar{y} = \sqrt{\frac{1}{l}\int_0^{2l} (f(x))^2 dx} = \sqrt{\frac{1}{1/2}\int_0^{1} \left(x^2\right)^2 dx}$$

$$= \sqrt{2\left[\frac{x^5}{5}\right]_0^1} = \sqrt{\frac{2}{5}}$$

PART-B

1. Expand $f(x) = \begin{cases} x & (0,\pi) \\ 2\pi - x & (\pi, 2\pi) \end{cases}$ as Fourier series and hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} .... = \frac{\pi^2}{8}$
2. Find the Fourier series for f(x) = x^2 in (-π . π) and also prove that

   (i) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} .... = \frac{\pi^2}{6}$ (ii) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - .... = \frac{\pi^2}{12}$
3. Expand f(x) = | cosx | as Fourier series in (-π . π).
4. Expand f(x) = xsinx as a Fourier series in $(0, 2\pi)$
5. Expand f(x) = |x| as a Fourier series in (-π . π) and deduce to $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} .... = \frac{\pi^2}{8}$
6. If $f(x) = \begin{cases} 0 \quad , & (-\pi, 0) \\ \sin x \,, & (0, \pi) \end{cases}$ Find the Fourier series and hence deduce that

   $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - .... = \frac{\pi - 2}{4}$
7. Find the Fourier expansion of $f(x) = (\pi - x)^2$ in $(0, 2\pi)$ and Hence deduce that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} .... = \frac{\pi^2}{6}$
8. Find a Fourier series to represent $f(x) = 2x - x^2$ with period 3 in the range $(0,3)$
9. Find the Fourier series of $f(x) = e^x$ in $(-\pi, \pi)$.

10. Find the Fourier series for $f(x)=\begin{cases}1 & in \quad (0,\pi)\\ 2 & in \quad (\pi,2\pi)\end{cases}$ and hence s.t $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}.........=\frac{\pi^2}{8}$

11. Find the the half range sine series for $f(x)=x(\pi-x)$ in the interval $(0,\pi)$ and deduce that $\frac{1}{1^3}-\frac{1}{3^3}-\frac{1}{5^3}....$

12. Obtain the half range cosine series for $f(x)=(x-1)^2$ in $(0,1)$ and also deduce that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}.........=\frac{\pi^2}{6}$

13. Find the Fourier series for f(x) = x^2 in ($-\pi . \pi$) and also prove that $1+\frac{1}{1^4}+\frac{1}{2^4}.........=\frac{\pi^4}{90}$

14. Find the Fourier series up to second harmonic

X	0	1	2	3	4	5
Y	9	18	24	28	26	20

15. Find the Fourier series up to third harmonic

X	0	π/3	2π/3	π	4π/3	5π/3	2π
F(x)	10	14	19	17	15	12	10

UNIT-III APPLICATIONS OF PARTIAL DIFFERENTIAL EQUATIONS

PART-A

1. Classify the Partial Differential Equation i) $\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial y^2}$

Answer:

$\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial y^2}$ here A=1,B=0,&C=-1

B^2 - 4AC=0-4(1)(-1)=4>0. The Partial Differential Equation is hyperbolic.

2. Classify the Partial Differential Equation $\frac{\partial^2 u}{\partial x\partial y}=\left(\frac{\partial u}{\partial y}\right)\left(\frac{\partial u}{\partial x}\right)+xy$

Answer:

$\frac{\partial^2 u}{\partial x\partial y}=\left(\frac{\partial u}{\partial y}\right)\left(\frac{\partial u}{\partial x}\right)+xy$ here A=0,B=1,&C=0

B^2-4AC=1-4(0)(0)=1>0. The Partial Differential Equation is hyperbolic.

3. Classify the following second order Partial Differential equation

$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial x}\right)^2$

Answer: $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial x}\right)^2$ here A=1,B=0,&C=1

B^2-4AC=0-4(1)(1)=-4<0 . The Partial Differential Equation is Elliptic.

4. Classify the following second order Partial Differential equation

$4\frac{\partial^2 u}{\partial x^2}+4\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 u}{\partial y^2}-6\left(\frac{\partial u}{\partial x}\right)-8\left(\frac{\partial u}{\partial y}\right)=0$

Answer:

$4\frac{\partial^2 u}{\partial x^2}+4\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 u}{\partial y^2}-6\left(\frac{\partial u}{\partial x}\right)-8\left(\frac{\partial u}{\partial y}\right)=0$

here A= 4,B =4, & C = 1

B^2-4AC =16 -4(4)(1) = 0. The Partial Differential Equation is Parabolic.

5. In the wave equation $\frac{\partial^2 y}{\partial t^2}=c^2\frac{\partial^2 y}{\partial x^2}$ **what does c^2 stands for?**

Answer:

$$\frac{\partial^2 y}{\partial t^2}=c^2\frac{\partial^2 y}{\partial x^2} \quad \text{here} \quad a^2=\frac{T}{m} \quad \text{T-Tension and m- Mass}$$

6. In one dimensional heat equation $u_t = \alpha^2 u_{xx}$ what does α^2 stands for?

Answer:- $$\frac{\partial u}{\partial t}=\alpha^2\frac{\partial^2 u}{\partial x^2}$$

$\alpha^2 = \dfrac{k}{\rho c}$ is called diffusivity of the substance

Where k – Thermal conductivity

ρ - Density

c – Specific heat

7. State any two laws which are assumed to derive one dimensional heat equation

Answer: i) Heat flows from higher to lower temp

ii) The rate at which heat flows across any area is proportional to the area and to the temperature gradient normal to the curve. This constant of proportionality is known as the conductivity of the material. It is known as Fourier law of heat conduction

8. A tightly stretched string of length 2ℓ is fastened at both ends. The midpoint of the string is displaced to a distance 'b' and released from rest in this position. Write the initial conditions.

Answer:

(i) y(0 , t) = 0

(ii) y(2ℓ ,t) = 0

(iii) $\left(\dfrac{\partial y}{\partial t}\right)_{t=0}=0$

(iv) y(x , 0) = $\begin{cases} \dfrac{b}{\ell}x & 0\le x\le \ell \\ \dfrac{b}{\ell}(2\ell-x) & \ell\le x\le 2\ell \end{cases}$

9. What are the possible solutions of one dimensional Wave equation?

Answer:

The possible solutions are

$y(x,t) = (A e^{\lambda x} + B e^{-\lambda x}) (C e^{\lambda at} + D e^{-\lambda at})$

$y(x,t) = (A \cos \lambda x + B \sin \lambda x)(C \cos \lambda at + D \sin \lambda at)$

$y(x,t) = (Ax + B) (Ct + D)$

10. What are the possible solutions of one dimensional head flow equation?

Answer:

The possible solutions are

$$u(x,t)=(Ae^{\lambda x}+Be^{-\lambda x})Ce^{-\alpha^2\lambda^2 t}$$

$$u(x,t)=(A\cos\lambda x+B\sin\lambda x)Ce^{-\alpha^2\lambda^2 t}$$

$$u(x,t)=(Ax+B)C$$

11. State Fourier law of heat conduction

Answer: $$Q=-kA\frac{\partial u}{\partial x}$$

(the rate at which heat flows across an area A at a distance from one end of a bar is proportional to temperature gradient)

Q=Quantity of heat flowing

k – Thermal conductivity, A=area of cross section ; $\dfrac{\partial u}{\partial x}$ =Temperature gradient

12. What are the possible solutions of two dimensional heat flow equation?

Answer:

The possible solutions are

$$u(x,y) = (Ae^{\lambda x} + Be^{-\lambda x})(C\cos\lambda y + D\sin\lambda y)$$
$$u(x,y) = (A\cos\lambda x + B\sin\lambda x)(Ce^{\lambda y} + De^{-\lambda y})$$
$$u(x,y) = (Ax+B)(Cy+D)$$

13. The steady state temperature distribution is considered in a square plate with sides x = 0 ,y = 0 , x = a and y = a. The edge y = 0 is kept at a constant temperature T and the three edges are insulated. The same state is continued subsequently. Express the problem mathematically.

Answer:

U(0,y) = 0 , U(a,y) = 0 ,U(x,a) = 0, U(x,0) = T.

PART-B

1. A tightly stretched string with fixed end points x = 0 and x = l is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity 3x (l-x). Find the displacement.
2. A string is stretched and fastened to two points and apart. Motion is started by displacing the string into the form y = K(lx-x^2) from which it is released at time t = 0. Find the displacement at any point of the string.
3. A string of length 2l is fastened at both ends. The midpoint of string is taken to a height b and then released from rest in that position. Find the displacement of the string.
4. A tightly stretched string with fixed end points x = 0 and x = l is initially at rest in its position given by y(x, 0) = $y_0 \sin^3 \frac{\pi x}{l}$. If it is released from rest find the displacement.
5. A string is stretched between two fixed points at a distance 2l apart and points of the string are given initial velocities where $V = \begin{cases} \frac{cx}{l} & 0 < x < 1 \\ \frac{c}{l}(2l-x) & 0 < x < 1 \end{cases}$. Find the displacement.
6. Derive all possible solution of one dimensional wave equation. Derive all possible solution of one dimensional heat equation. Derive all possible solution of two dimensional heat equations.
7. A rod 30 cm long has its end A and B kept at 20°C and 80°C, respectively until steady state condition prevails. The temperature at each end is then reduced to 0°C and kept so. Find the resulting temperature u(x, t) taking x = 0.
8. A rectangular plate with insulated surface is 8 cm wide so long compared to its width that itA bar 10 cm long , with insulated sides has its end A & B kept at 20°C and 40°C respectively until the steady state condition prevails. The temperature at A is suddenly raised to 50°C and B is lowered to 10°C. Find the subsequent temperature function u(x , t).
9. A rectangular plate with insulated surface is 8 cm wide and so long compared to its width that it may be considered as an infinite plate. If the temperature along short edge y = 0 is u (x ,0) = $100\sin\frac{\pi x}{8}$, 0<x<8 While two edges x = 0 and x = 8 as well as the other short edges are kept at 0°C. Find the steady state temperature.
10. A rectangular plate with insulated surface is10 cm wide so long compared to its width that it may be considered as an infinite plate. If the temperature along short edge y = 0 is given by $u = \begin{cases} 20x & 0 \le x \le 5 \\ 20(10-x) & 5 \le x \le 10 \end{cases}$ and all other three edges are kept at 0° C. Find the steady state temperature at any point of the plate.

UNIT-IV FOURIER TRANSFORMS

PART-A

1. **State Fourier Integral Theorem.**

Answer:

If $f(x)$ is piece wise continuously differentiable and absolutely on $(-\infty,\infty)$ then,

$$f(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t)e^{i(x-t)s}dt\ ds.$$

2. **State and prove Modulation theorem.**

$$F\left[f(x)\cos ax\right]=\frac{1}{2}\left[F(s+a)+F(s-a)\right]$$

Proof:

$$F\left[f(x)\cos ax\right]=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\cos ax\ e^{isx}dx$$

$$=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\left(\frac{e^{iax}+e^{-iax}}{2}\right)e^{isx}dx$$

$$=\frac{1}{2}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\ e^{i(s+a)x}dx+\frac{1}{2}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\ e^{i(s-a)x}dx$$

$$=\frac{1}{2}F(s+a)+\frac{1}{2}F(s-a)$$

$$F\left[f(x)\cos ax\right]=\frac{1}{2}\left[F(s+a)+F(s-a)\right]$$

3. **State Parseval's Identity.**

Answer:

If $F(s)$ is a Fourier transform of $f(x)$, then

$$\int_{-\infty}^{\infty}|F(s)|^2\,ds=\int_{-\infty}^{\infty}|f(x)|^2\,dx$$

4. **State Convolution theorem.**

Answer:

The Fourier transforms of Convolution of $f(x)$ and $g(x)$ is the product of their Fourier transforms. $F\{f*g\}=F(s)G(s)$

5. **State and prove Change of scale of property.**

Answer:

_If $F(s)=F\{f(x)\}$, then $F\{f(ax)\}=\frac{1}{a}F(s/a)$

$$F\{f(ax)\}=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(ax)e^{isx}dx$$

$$=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)e^{i(s/a)t}\frac{dt}{a};\quad \text{where } t=ax\ ,\qquad F\{f(ax)\}=\frac{1}{a}F\left(s/a\right)$$

6. **Prove that if F[f(x)] = F(s) then** $F\{x^n f(x)\}=(-i)^n\frac{d^n}{ds^n}F(s)$

Answer:

$$F(s)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{isx}dx$$, Diff w.r.t s '*n*' times

$$\frac{d^n}{ds^n}F(s)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)(ix)^n e^{isx}dx$$

$$=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)(i)^n(x)^n e^{isx}dx$$

$$\frac{1}{(i)^n}\frac{d^n}{ds^n}F(s)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}(x)^n f(x)e^{isx}dx$$

$$(-i)^n\frac{d^n}{ds^n}F(s)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}(x)^n f(x)e^{isx}dx$$

$$F\left[x^n f(x)\right]=(-i)^n\frac{d^n}{ds^n}F(s)$$

7. Solve for f(x) from the integral equation $\int_0^{\infty} f(x)\cos sx dx = e^{-s}$

Answer:

$$\int_0^{\infty} f(x)\cos sx dx = e^{-s}$$

$$F_c\left[f(x)\right]=\sqrt{\frac{2}{\pi}}\int_0^{\infty} f(x)\cos sx\, dx$$

$$F_c\left[f(x)\right]=\sqrt{\frac{2}{\pi}}\; e^{-s}$$

$$f(x)=\sqrt{\frac{2}{\pi}}\int_0^{\infty}F_c\left[f(x)\right]\cos sx\, ds$$

$$=\sqrt{\frac{2}{\pi}}\int_0^{\infty}\sqrt{\frac{2}{\pi}}\; e^{-s}\cos sx\, ds$$

$$=\frac{2}{\pi}\int_0^{\infty} e^{-s}\cos sx\, ds \qquad =\frac{2}{\pi}\left[\frac{1}{x^2+1}\right]$$

$$\int_0^{\infty} e^{-ax}\cos bx\, dx=\frac{a}{a^2+b^2}$$

$$\therefore a=1,\ b=x$$

8. Find the complex Fourier Transform of $f(x)=\begin{cases}1 & |x|<a\\ 0 & |x|>a>0\end{cases}$

Answer: $F\left[f(x)\right]=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{isx}dx$ $\quad \left|\because |x|<a;\ -a<x<a\right.$

$$F\left[f(x)\right]=\frac{1}{\sqrt{2\pi}}\int_{-a}^{a}(1)e^{isx}dx$$

$$=\frac{1}{\sqrt{2\pi}}\int_{-a}^{a}(\cos sx+i\sin sx)dx$$

$$=\frac{2}{\sqrt{2\pi}}\int_{0}^{a}(\cos sx)dx=\frac{2}{\sqrt{2\pi}}\left(\frac{\sin sx}{s}\right)_{0}^{a}$$

$$=\sqrt{\frac{2}{\pi}}\left(\frac{\sin as}{s}\right)$$

[Use even and odd property second term become zero]

9. Find the complex Fourier Transform of $f(x)=\begin{cases} x & |x|<a \\ 0 & |x|>a>0 \end{cases}$

Answer:

$$F\left[f(x)\right]=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{isx}dx$$

$$=\frac{1}{\sqrt{2\pi}}\int_{-a}^{a} x\,e^{isx}dx \qquad \left|\because |x|<a;\ -a<x<a\right.$$

$$=\frac{1}{\sqrt{2\pi}}\int_{-a}^{a} x(\cos sx+i\sin sx)dx$$

$$=\frac{2}{\sqrt{2\pi}}\int_{0}^{a}(x(i\sin sx)dx=\frac{2i}{\sqrt{2\pi}}\left[x\left(\frac{-\cos sx}{s}\right)-(1)\left(\frac{-\sin sx}{s^2}\right)\right]_{0}^{a}$$

$$=i\sqrt{\frac{2}{\pi}}\left[\frac{-as\cos as+\sin as}{s^2}\right]$$

[Use even and odd property first term become zero]

10.Write Fourier Transform pair.

Answer: If $f(x)$ is defined in $(-\infty,\infty)$, then its Fourier transform is defined as

$F(s)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{isx}dx$, If $F(s)$ is an Fourier transform of $f(x)$, then at every point of Continuity of $f(x)$, we have $f(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F(s)e^{-isx}ds$.

11. Find the Fourier cosine Transform of f(x) = e^{-x}

Answer:

$$F_c\left[f(x)\right]=\sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\cos sx\,dx$$

$$F_c\left[e^{-x}\right]=\sqrt{\frac{2}{\pi}}\int_0^\infty e^{-x}\cos sx\,dx \qquad \because \int_0^\infty e^{-ax}\cos bx\;dx=\frac{a}{a^2+b^2}$$

$$F_c\left[e^{-x}\right]=\sqrt{\frac{2}{\pi}}\;\frac{1}{s^2+1}$$

12.Find the Fourier Transform of $f(x)=\begin{cases} e^{imx} & ,a<x<b \\ 0, & otherwise \end{cases}$

Answer:

$$F\left[f(x)\right]=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{isx}dx$$

$$=\frac{1}{\sqrt{2\pi}}\int_a^b e^{imx}e^{isx}dx \quad =\frac{1}{\sqrt{2\pi}}\int_a^b e^{i(m+s)x}dx$$

$$=\frac{1}{\sqrt{2\pi}}\left[\frac{e^{i(m+s)x}}{i(m+s)}\right]_a^b \quad =\frac{1}{\sqrt{2\pi}}\frac{1}{i(m+s)}\left[e^{i(m+s)b}-e^{i(m+s)a}\right]$$

13.Find the Fourier sine Transform of $\frac{1}{x}$**.**

Answer:

$$F_s\left[f(x)\right]=\sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\sin sx\,dx$$

$$=\sqrt{\frac{2}{\pi}}\int_0^\infty \frac{\sin sx}{x}\;dx \;=\sqrt{\frac{2}{\pi}}\;\frac{\pi}{2}$$

$$F_s\left[\frac{1}{x}\right]=\sqrt{\frac{\pi}{2}}$$

14. Find the Fourier sine transform of e^{-x}

$$F_s\left[f(x)\right]=\sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\sin sx\,dx$$

$$F_s\left[e^{-x}\right]=\sqrt{\frac{2}{\pi}}\int_0^\infty e^{-x}\sin sx\,dx \qquad \because \int_0^\infty e^{-ax}\sin bx\;dx=\frac{b}{a^2+b^2}$$

$$F_s\left[e^{-x}\right]=\sqrt{\frac{2}{\pi}}\;\frac{s}{s^2+1}$$

15. Find the Fourier cosine transform of $e^{-2x}+2e^{-x}$

Answer: $F_c\left[f(x)\right]=\sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\cos sx\;dx$

$$F_c\left[e^{-2x}+2e^{-x}\right]=\sqrt{\frac{2}{\pi}}\int_0^\infty\left(e^{-2x}+2e^{-x}\right)\cos sx\,dx$$
$$=\sqrt{\frac{2}{\pi}}\left\{\int_0^\infty e^{-2x}\cos sx\,dx+2\int_0^\infty e^{-x}\cos sx\,dx\right\}$$
$$=\sqrt{\frac{2}{\pi}}\left\{\frac{2}{s^2+4}+2\frac{1}{s^2+1}\right\}=2\sqrt{\frac{2}{\pi}}\left\{\frac{1}{s^2+4}+\frac{1}{s^2+1}\right\}$$

PART-B

1. Find the Fourier Transform of $f(x)=\begin{cases}1-x^2 & if\ |x|<1\\ 0 & if\ |x|>1\end{cases}$ and hence

   deduce that (i) $\int_0^\infty\left(\frac{x\cos x-\sin x}{x^3}\right)\cos\left(\frac{x}{2}\right)dx=-\frac{3\pi}{16}$ (ii) $\int_0^\infty\left(\frac{\sin x-x\cos x}{x^3}\right)^2dx=\frac{\pi}{15}$

2. Find the Fourier Transform of $f(x)=\begin{cases}a^2-x^2 & |x|<a\\ 0 & |x|>a>0\end{cases}$. hence S.T $\int_0^\infty\left(\frac{\sin x-x\cos x}{x^3}\right)dx=\frac{\pi}{4}$

3.Find the Fourier Transform of $f(x)=\begin{cases}1 & if\ |x|<a\\ 0 & if\ |x|>a\end{cases}$ and hence evaluate

   $i)\int_0^\infty\frac{\sin x}{x}dx$ ii) $\int_0^\infty\left(\frac{\sin x}{x}\right)^2dx$

4. Find Fourier Transform of $f(x)=\begin{cases}1-|x| & if\ |x|<1\\ 0 & if\ |x|>1\end{cases}$ and hence evaluate

   i) $\int_0^\infty\left(\frac{\sin x}{x}\right)^2dx$ ii) $\int_0^\infty\left(\frac{\sin x}{x}\right)^4dx$

5. Evaluate i) $\int_0^\infty\frac{x^2}{\left(x^2+a^2\right)^2}dx$ ii) $\int_0^\infty\frac{dx}{\left(x^2+a^2\right)^2}$

6. Evaluate (a) $\int_0^\infty\frac{dx}{\left(x^2+1\right)\left(x^2+4\right)}$ (b) $\int_0^\infty\frac{t^2dt}{\left(t^2+4\right)\left(t^2+9\right)}$

7. Find the Fourier sine transform of $f(x)=\begin{cases}\sin x;when & o<x<\pi\\ 0 & ;whenx>\pi\end{cases}$

8. Find the Fourier cosine transform of $f(x)=\begin{cases}\cos x;when & o<x<a\\ 0 & ;whenx>a\end{cases}$

9. Show that Fourier transform $e^{\frac{-x^2}{2}}$ is $e^{\frac{-s^2}{2}}$

10.Obtain Fourier cosine Transform of $e^{-a^2x^2}$ and hence find Fourier sine Transform $x\,e^{-a^2x^2}$

11. Solve for f(x) from the integral equation $\int_0^\infty f(x)\cos\alpha x\,dx=e^{-\alpha}$

12.Solve for f(x) from the integral equation $\int_0^\infty f(x)\sin tx\,dx = \begin{cases} 1 & ,0 \le t < 1 \\ 2 & ,1 \le t < 2 \\ 0 & ,t \ge 2 \end{cases}$

13.Find Fourier sine Transform of e^{-x}, x>0 and hence deduce that $\int_0^\infty \frac{x\sin x}{\left(1+x^2\right)}dx$

UNIT-V Z- TRANSFORMS

PART-A

1. Define Z transform

Answer:

Let {f(n)} be a sequence defined for n = 0, 1,2 … and f(n) = 0 for n< 0 then its Z – Transform is defined as

$$Z\left[f(n)\right] = F(z) = \sum_{n=-\infty}^{\infty} f(n) z^{-n} \quad \text{(Two sided z transform)}$$

$$Z\left[f(n)\right] = F(z) = \sum_{n=0}^{\infty} f(n) z^{-n} \quad \text{(One sided z transform)}$$

2. Find the Z Transform of 1

Answer: $Z\left[f(n)\right] = \sum_{n=0}^{\infty} f(n) z^{-n}$

$$Z[1] = \sum_{n=0}^{\infty} (1) z^{-n} = 1 + z^{-1} + z^{-2} + \ldots. = \left(1 - z^{-1}\right)^{-1}$$

$$= \left(1 - \frac{1}{z}\right)^{-1} = \left(\frac{z-1}{z}\right)^{-1} = \left(\frac{z}{z-1}\right) \qquad Z(1) = \left(\frac{z}{z-1}\right)$$

3. Find the Z Transform of n

Answer:

$$Z\left[f(n)\right] = \sum_{n=0}^{\infty} f(n) z^{-n}$$

$$Z(n) = \sum_{n=0}^{\infty} n z^{-n}$$

$$= \sum_{n=0}^{\infty} n z^{-n} = 0 + z^{-1} + 2z^{-2} + 3z^{3} + \ldots$$

$$= z^{-1}\left(1 - z^{-1}\right)^{-2} = \frac{1}{z}\left[1 - \frac{1}{z}\right]^{-2} = \frac{1}{z}\left[\frac{z}{z-1}\right]^{2}$$

$$= \frac{z}{(z-1)^2}$$

4. Find the Z Transform of n².

Answer: $Z\left(n^2\right) = Z(n\,n) = -z\frac{d}{dz}\left(Z(n)\right)$, by the property,

$$= -z\frac{d}{dz}\left(\frac{z}{(z-1)^2}\right) = (-z)\left[\frac{(z-1)^2 - z2(z-1)}{(z-1)^4}\right] = \frac{z^2+z}{(z-1)^3}$$

5. State Initial & Final value theorem on Z Transform

Initial Value Theorem

If Z [f (n)] = F (z) then f (0) = $\lim_{z\to\infty} F(z)$

Final Value Theorem

If Z [f (n)] = F (z) then $\lim_{n\to\infty} f(n) = \lim_{z\to 1}(z-1)F(z)$

6. State convolution theorem of Z- Transform.

Answer:

Z[f(n)] = F(z) and Z[g(n)] = G(z) then Z{f(n)*g(n)} = F(z) · G(z)

7. Find Z –Transform of na^n

Answer:

$$Z\left[f(n)\right] = \sum_{n=0}^{\infty} f(n) z^{-n}$$

$$Z\left(na^n\right) = \sum_{n=0}^{\infty}\left(na^n\right)z^{-n}$$

$$= \sum_{n=0}^{\infty} n\left(\frac{a}{z}\right)^n = 0 + \left(\frac{a}{z}\right)^1 + 2\left(\frac{a}{z}\right)^2 + 3\left(\frac{a}{z}\right)^3 + \ldots$$

$$= \frac{a}{z}\left(1-\frac{a}{z}\right)^{-2} = \frac{az}{(z-a)^2}$$

8. Find Z – Transform of $\cos\frac{n\pi}{2}$ and $\sin\frac{n\pi}{2}$

Answer: We know that $Z\left[f(n)\right] = \sum_{n=0}^{\infty} f(n) z^{-n}$

$$Z\left[\cos n\theta\right] = \frac{z(z-\cos\theta)}{z^2 - 2z\cos\theta + 1}$$

=>> $$Z\left[\cos n\frac{\pi}{2}\right] = \frac{z\left(z-\cos\frac{\pi}{2}\right)}{z^2 - 2z\cos\frac{\pi}{2} + 1} = \left(\frac{z^2}{z^2+1}\right)$$

Similarly $$Z\left[\sin n\theta\right] = \frac{z\sin\theta}{z^2 - 2z\cos\theta + 1}$$

=>> $$Z\left[\sin n\frac{\pi}{2}\right] = \frac{z\sin\frac{\pi}{2}}{z^2 - 2z\cos\frac{\pi}{2} + 1} = \frac{z}{z^2+1}$$

9. Find Z – Transform of $\left(\frac{1}{n}\right)$

Answer: $$Z\left[f(n)\right] = \sum_{n=0}^{\infty} f(n) z^{-n}$$

$$Z\left(\frac{1}{n}\right)=\sum_{n=0}^{\infty}\frac{1}{n}z^{-n}$$

$$=\sum_{n=1}^{\infty}\frac{1}{n}z^{-n}=\frac{z^{-1}}{1}+\frac{z^{-2}}{2}+\frac{z^{-3}}{3}+....$$

$$=-\log\left(1-\frac{1}{z}\right)=\log\left(\frac{z-1}{z}\right)^{-1}$$

$$=\log\left(\frac{z}{z-1}\right)$$

10. Find Z – Transform of $\left(\frac{1}{n!}\right)$

Answer:

$$Z\left[f(n)\right]=\sum_{n=0}^{\infty}f(n)z^{-n}$$

$$Z\left(\frac{1}{n!}\right)=\sum_{n=0}^{\infty}\frac{1}{n!}z^{-n}$$

$$\sum_{n=0}^{\infty}\frac{1}{n!}z^{-n}=1+\frac{z^{-1}}{1!}+\frac{z^{-2}}{2!}+\frac{z^{-3}}{3!}+....$$

$$=e^{z^{-1}}=e^{\frac{1}{z}}$$

11. Find Z – Transform of $\frac{1}{n+1}$

Answer:

$$Z\left[f(n)\right]=\sum_{n=0}^{\infty}f(n)z^{-n}$$

$$Z\left(\frac{1}{n+1}\right)=\sum_{n=0}^{\infty}\frac{1}{n+1}z^{-n}$$

$$=z\sum_{n=0}^{\infty}\frac{1}{n+1}z^{-(n+1)}$$

$$=z\left[z^{-1}+\frac{z^{-2}}{2}+\frac{z^{-3}}{3}+....\right]$$

$$=z\left(-\log\left(1-\frac{1}{z}\right)\right)$$

$$=z\log\left(\frac{z}{z-1}\right)$$

12. State and prove First shifting theorem

Statement: If $Z\left(f\left(t\right)\right)=F\left(z\right)$, then $Z\left(e^{-at}f(t)\right)=F\left(ze^{aT}\right)$

Proof:

$$Z\left(e^{-at}f(t)\right)=\sum_{n=0}^{\infty}e^{-anT}f(nT)z^{-n}$$

As f(t) is a function defined for discrete values of t, where t = nT,

then the Z-transform is

$$Z\left(f(t)\right)=\sum_{n=0}^{\infty}f(nT)z^{-n}=F(z).$$

$$Z\left(e^{-at}f(t)\right)=\sum_{n=0}^{\infty}f(nT)\left(ze^{aT}\right)^{-n}=F(ze^{aT})$$

13. Define unit impulse function and unit step function.

Answer:

The unit sample sequence is defined as follows:

$$\delta(n)=\begin{cases}1 & for\, n=0\\ 0 & for\, n\neq 0\end{cases}$$

The unit step sequence is defined as follows:

$$u(n)=\begin{cases}1 & for\, n\geq 0\\ 0 & for\, n<0\end{cases}$$

14. Find Z – Transform of $Z\left(e^{at}\right)$

Answer:

$$Z\left(e^{at}\right)=\sum_{n=0}^{\infty}e^{anT}z^{-n}=\sum_{n=0}^{\infty}\left(e^{aT}\right)^{n}z^{-n}=z\left(e^{aT}\right)^{n}$$

$$=\frac{z}{z-e^{aT}} \qquad \left[z\left(a^{n}\right)=\frac{z}{z-a}\right]$$ [Using First shifting theorem]

15. Find Z – Transform of $Z\left(te^{-2t}\right)$

Answer:

$$Z\left(te^{-2t}\right)=Z\left(t\right)_{z=ze^{2T}}=\left(\frac{Tz}{\left(z-1\right)^{2}}\right)_{z=ze^{2T}}$$

$$=\left(\frac{Tze^{2T}}{\left(ze^{2T}-1\right)^{2}}\right)$$ [Using First shifting theorem]

16. Find Z – Transform of $Z\left(e^{t}\cos 2t\right)$

Answer: $$Z\left(e^{t}\cos 2t\right)=Z\left(\cos 2t\right)_{z=ze^{-T}}=\left(\frac{z\left(z-\cos\theta\right)}{z^{2}-\left(2\cos\theta\right)z+1}\right)_{z=ze^{-T}}$$

$$=\left(\frac{ze^{-T}\left(ze^{-T}-\cos T\right)}{ze^{-2T}-\left(2\cos T\right)ze^{-T}+1}\right)$$ [Using First shifting theorem]

17. Find Z – Transform of $Z\left(\sin\left(t+T\right)\right)$

Answer: Let f (t) = sin$t$, by second sifting theorem

$$Z\left(\sin(t+T)\right)=Z\left(f(t+T)\right)=z\left(F(z)-f(0)\right)$$

$$=z\left(\frac{z\sin t}{z^2-\left(2\cos t\right)z+1}-0\right)=\left(\frac{z^2\sin t}{z^2-\left(2\cos t\right)z+1}\right)$$

18. Find Z – transform of $(n+1)(n+2)$

Answer: $Z\left[f(n)\right]=\sum_{n=0}^{\infty} f(n)z^{-n}$

$$Z\left[(n+1)(n+2)\right]=Z\left[n^2+2n+n+2\right]$$

$$=Z\left[n^2+3n+2\right]=z\left[n^2\right]+3z\left[n\right]+2z\left[1\right]$$

$$=\frac{z^2+z}{\left(z-1\right)^3}+3\frac{z}{\left(z-1\right)^2}+2\frac{z}{z-1}$$

PART-B

1. Find (i) $Z^{-1}\left[\frac{8z^2}{(2z-1)(4z+1)}\right]$ (ii) $Z^{-1}\left[\frac{8z^2}{(2z-1)(4z-1)}\right]$ by convolution theorem.
2. Find (i) $Z^{-1}\left[\frac{z^2}{(z-a)(z-b)}\right]$ (ii) $Z^{-1}\left[\frac{z^2}{(z-1)(z-3)}\right]$ by convolution theorem
3. Find (i) $Z^{-1}\left[\frac{z^2}{(z+a)^2}\right]$ (ii) $Z^{-1}\left[\frac{z^2}{(z-a)^2}\right]$ by convolution theorem
4. State and prove Initial & Final value theorem.
5. State and prove Second shifting theorem
6. Find the Z transform of (i) $\left(\frac{1}{(n+1)(n+2)}\right)$ (ii) $\left(\frac{2n+3}{(n+1)(n+2)}\right)$
7. Find $Z^{-1}\left[\frac{z^2}{(z^2+4)}\right]$ by residues.
8. Find the inverse Z transform of $\frac{z^2+z}{\left(z-1\right)(z^2+1)}$ by partial fractions.
9. Find (i) $Z^{-1}\left[\frac{z}{z^2-2z+2}\right]$ (ii) $Z^{-1}\left[\frac{z^2}{z^2-7z+10}\right]$
10. Find the Z transform of $f(n)=\frac{1}{n!}$ Hence find $Z\left[\frac{1}{(n+1)!}\right]$ and $Z\left[\frac{1}{(n+2)!}\right]$.
11. Find $Z\left[\frac{1}{n!}\right]$ and also find the value of $\sin(n+1)\theta$ and $\cos(n+1)\theta$.
12. Solve $y\left(n+2\right)+6y\left(n+1\right)+9y\left(n\right)=2^n \, with \; y\left(0\right)=0 \,\&\, y\left(1\right)=0$
13. Solve $y\left(n+2\right)-4y\left(n+1\right)+4y\left(n\right)=0$ y(0) = 1 ,y(1) =0
14. Solve $y\left(n\right)-3y\left(n-1\right)-4y\left(n-2\right)=0, n\geq 2 \quad given \; y(0)=3 \,\&\, y(1)=-2$
15. Solve $y\left(n+3\right)-3y\left(n+1\right)+2y\left(n\right)=0, \quad y\left(0\right)=4, y\left(1\right)=0 \,\&\, y\left(2\right)=8,$